

Answer all TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 The point A has coordinates $(5, -4)$
The point B has coordinates $(13, 1)$
- (a) Work out the coordinates of the midpoint of AB .

$$\frac{5+13}{2}, \quad \frac{-4+1}{2}$$

$$9, \quad -1.5$$

$$\left(\frac{9}{\dots\dots\dots}, \frac{-1.5}{\dots\dots\dots} \right)$$

(2)

Line L has equation $y = 2 - 3x$

- (b) Write down the gradient of line L .

$$y = -3x + 2$$

$$\frac{-3}{\dots\dots\dots}$$

(1)

Line L has equation $y = 2 - 3x$

- (c) Does the point with coordinates $(100, -302)$ lie on line L ?
You must give a reason for your answer.

$$\begin{matrix} (100, -302) \\ x \quad y \end{matrix}$$

$$y = 2 - 3 \times 100 = 2 - 300 = -298$$

No, it goes through $(100, -298)$

(1)

(Total for Question 1 is 4 marks)

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2 Find the lowest common multiple (LCM) of 28 and 105

$$28 = 2^2 \times 7$$

$$\begin{aligned} 105 &= 5 \times 21 \\ &= 5 \times 3 \times 7 \\ &= 3 \times 5 \times 7 \end{aligned}$$

$$\text{HCF} = 7$$

$$\begin{aligned} \text{LCM} &= 2^2 \times 3 \times 5 \times 7 \\ &= 420 \end{aligned}$$

420

(Total for Question 2 is 2 marks)

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3 The diagram shows a shape.

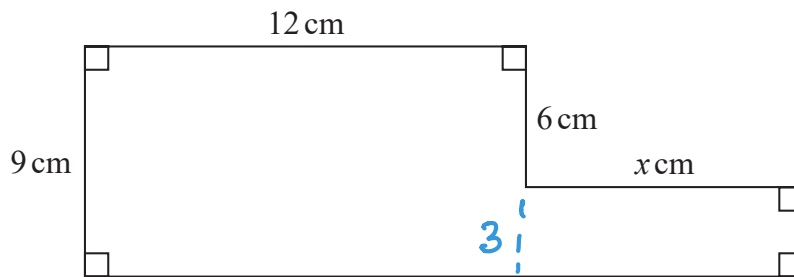


Diagram NOT
accurately drawn

The shape has area 129 cm^2

Work out the value of x .

$$12 \times 9 + 3x = 129$$

$$3x = 129 - 108$$

$$3x = 21$$

$$x = \frac{21}{3}$$

$$x = 7$$

(Total for Question 3 is 4 marks)



- 4 The table shows information about the weights, in kilograms, of 40 babies.

Weight (w kg)	Frequency
$2 < w \leq 3$	12
$3 < w \leq 4$	16
$4 < w \leq 5$	9
$5 < w \leq 6$	2
$6 < w \leq 7$	1

40

- (a) Write down the modal class.

$3 < w \leq 4$
(1)

- (b) Work out an estimate for the mean weight of the 40 babies.

$$\begin{aligned} & 2.5 \times 12 + 3.5 \times 16 + 4.5 \times 9 + 5.5 \times 2 + 6.5 \times 1 \\ & = 30 + 56 + 40.5 + 11 + 6.5 \\ & = 144 \end{aligned}$$

$$\text{Mean} = \frac{144}{40}$$

3.6 kg
(4)

One of the 40 babies is going to be chosen at random.

- (c) Find the probability that this baby has a weight of more than 5 kg.

$$> 5 = 2 + 1 = 3$$

$\frac{3}{40}$
(2)

(Total for Question 4 is 7 marks)



- 5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3:5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

$$120$$

$$G : B$$

$$3 : 5$$

$$120 \div 8 = 15$$

$$3 \times 15 = 45$$

$$5 \times 15 = 75$$

Sunday G = 45

$$\text{climbing} = 15$$

$$\text{sailing} = 30$$

B = 75

$$\text{climbing} = \frac{16}{25} \times 75$$

$$= 48$$

$$\text{sailing} = 75 - 48 = 27$$

$$\begin{aligned} \text{Sailing total} &= 30 + 27 \\ &= 57 \end{aligned}$$

57

(Total for Question 5 is 6 marks)



- 6 (a) Write 7.8×10^{-4} as an ordinary number.

$$0.00078$$

(1)

- (b) Work out $\frac{5.6 \times 10^4 + 7 \times 10^3}{2.8 \times 10^{-3}}$

Give your answer in standard form.

$$\frac{63000}{0.0028}$$

$$= 22500000$$

$$2.25 \times 10^7$$

(2)

(Total for Question 6 is 3 marks)

- 7 (a) Expand and simplify $(m - 8)(m + 5)$

$$m \times m = m^2$$

$$m \times 5 = 5m$$

$$-8 \times m = -8m$$

$$-8 \times 5 = -40$$

$$m^2 - 3m - 40$$

(2)

- (b) Factorise fully $5y + 20y^2$

$$5y(1 + 4y)$$

(2)



(c) Simplify $(p^2 + 3)^0$

1
.....
(1)

(d) Solve $3(2x - 5) = \frac{9 - x}{2}$

Show clear algebraic working.

x 2 $2(6x - 15) = 9 - x$

$$\begin{array}{r} 12x - 30 = 9 - x \\ +x \qquad \qquad +x \end{array}$$

$$\begin{array}{r} 13x - 30 = 9 \\ +30 \qquad +30 \end{array}$$

$$13x = 39$$

$$x = \frac{39}{13}$$

x = 3
(4)

(Total for Question 7 is 9 marks)

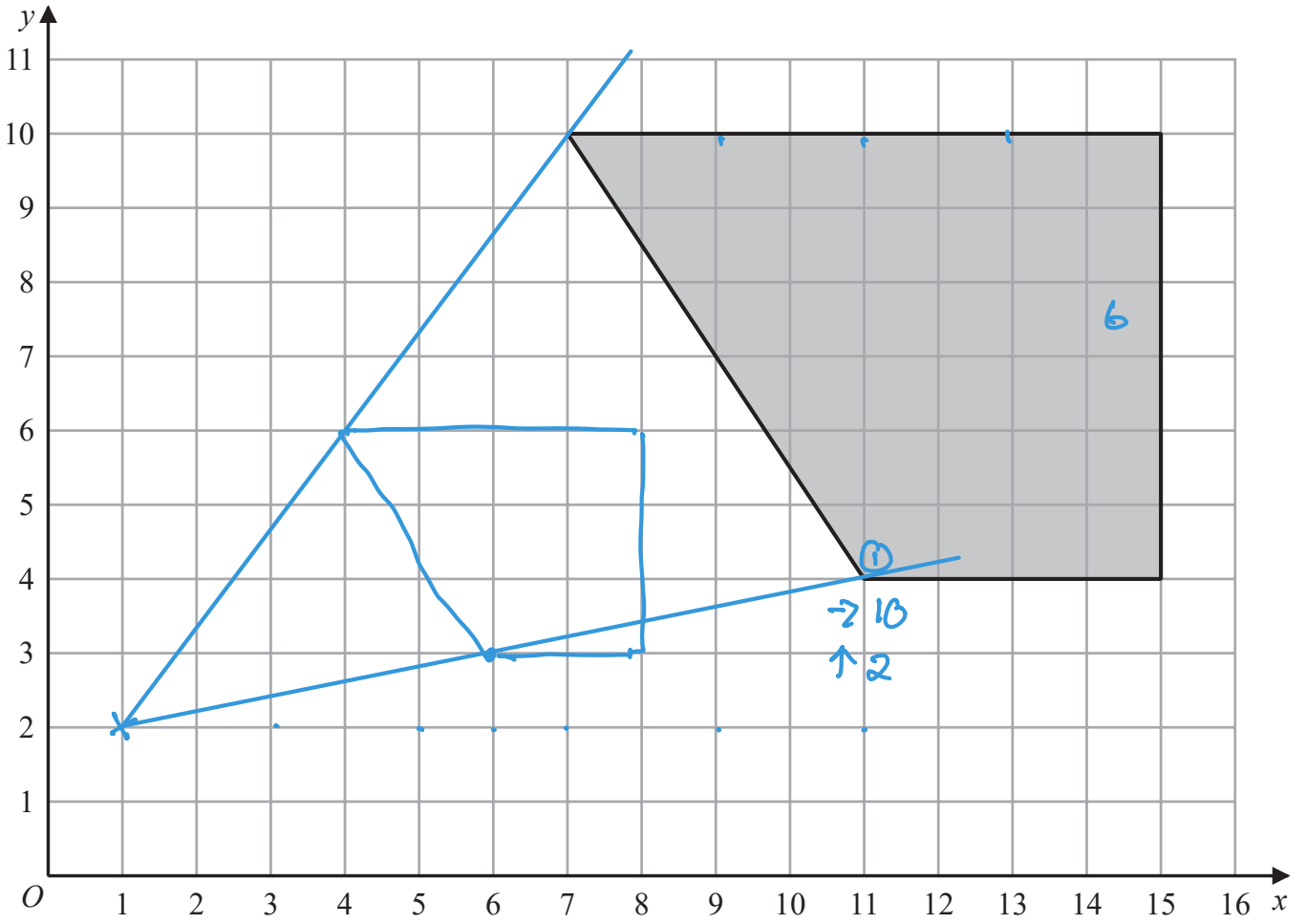
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8



On the grid, enlarge the shaded shape with scale factor $\frac{1}{2}$ and centre (1, 2)

① → 5 ↑ 1

(Total for Question 8 is 2 marks)

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9 Here is a right-angled triangle.

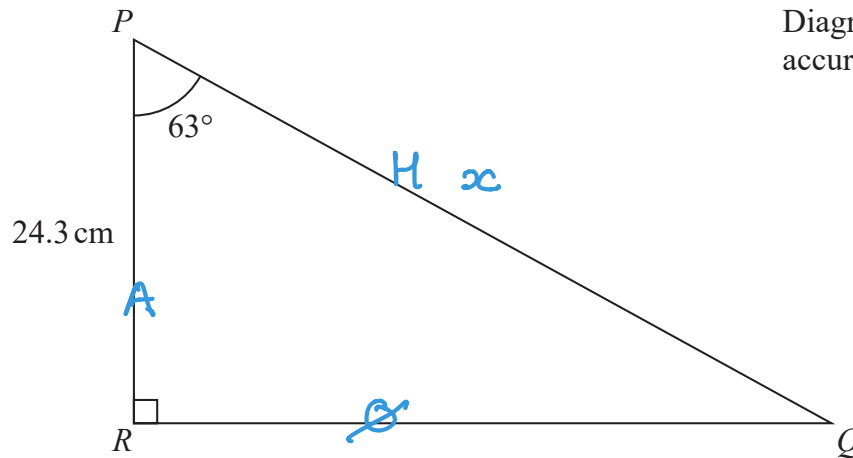


Diagram NOT
accurately drawn

Calculate the length of PQ . x
Give your answer correct to 3 significant figures.

$$\cos 63 = \frac{24.3}{x}$$

$$\begin{aligned} x &= \frac{24.3}{\cos 63} \\ &= 53.5253\dots \\ &\quad \uparrow \\ &\quad (3 \text{ s.f.}) \end{aligned}$$

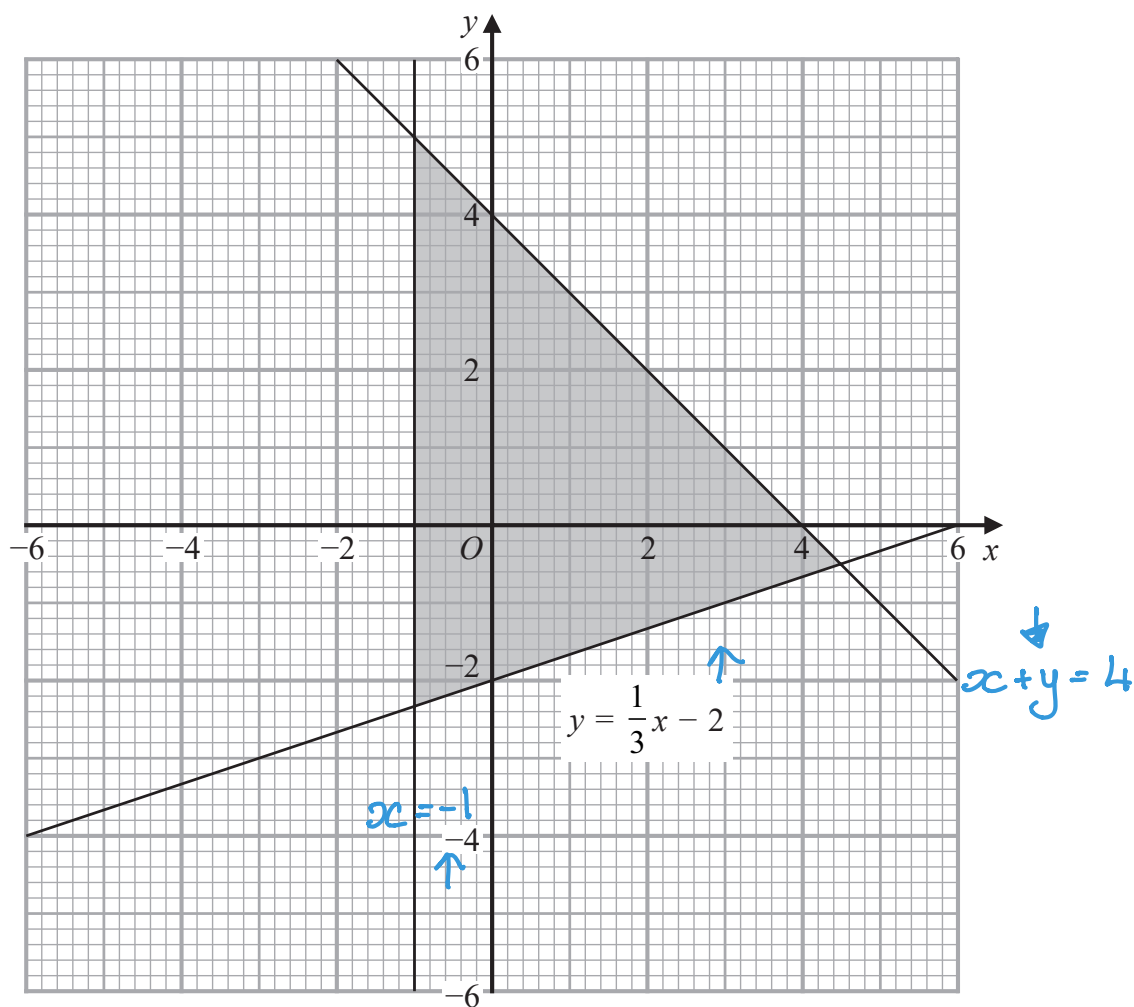
53.5

..... cm

(Total for Question 9 is 3 marks)



- 10 The shaded region in the diagram is bounded by three lines.
The equation of one of the lines is given.



Write down the three inequalities that define the shaded region.

$$x \geq -1$$

$$x + y \leq 4$$

$$y \geq \frac{1}{3}x - 2$$

(Total for Question 10 is 3 marks)



- 11 Max invests \$6000 in a savings account for 3 years.
The account pays compound interest at a rate of 1.5% per year for the first 2 years.

The compound interest rate changes for the third year.
At the end of 3 years, there is a total of \$6311.16 in the account.

Work out the compound interest rate for the third year.
Give your answer correct to 1 decimal place.

$$6000 \times 1.015 \times 1.015 \times x = 6311.16$$

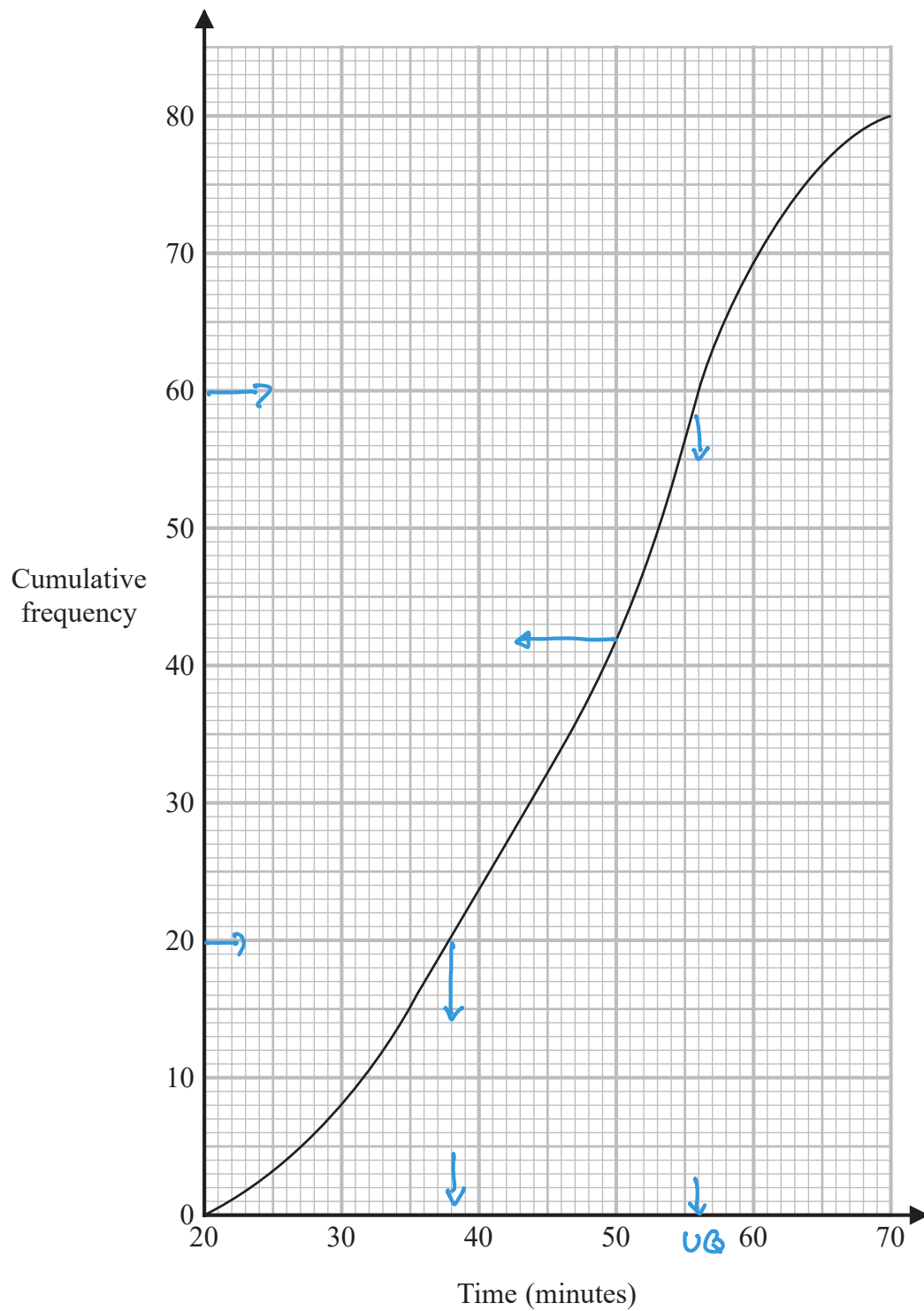
$$\begin{aligned} x &= \frac{6311.16}{6181.35} \\ &= 1.02100\dots \\ &\hookrightarrow 2.1\% \end{aligned}$$

..... 2.1 %

(Total for Question 11 is 3 marks)



- 12 A total of 80 men and women took part in a race.
The cumulative frequency graph gives information about the times, in minutes, they took for the race.



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(a) Use the graph to find an estimate for the interquartile range.

$$UQ = 56 \quad LQ = 38$$

$$IQR = 56 - 38$$

..... 18 minutes
(2)

60% of the men took 50 minutes or less for the race.

No women took 50 minutes or less for the race.

(b) Work out an estimate for the number of men who took part in the race.

$$50 \text{ or less} = 42$$

$$\begin{array}{l}
 60\% \text{ of men} = 42 \\
 \div 6 \quad \downarrow \quad \quad \quad = 7 \quad \downarrow \quad \div 6 \\
 10\% \quad \quad \quad \quad \quad = 70 \quad \downarrow \quad \times 10 \\
 \times 10 \quad \downarrow \quad 100\%
 \end{array}$$

..... 70
(3)

(Total for Question 12 is 5 marks)

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13 The diagram shows a solid cube.

The cube is placed on a table so that the whole of one face of the cube is in contact with the table.

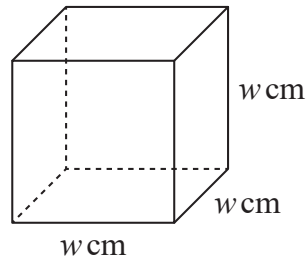


Diagram **NOT** accurately drawn

The cube exerts a force of 56 newtons on the table.

The pressure on the table due to the cube is 0.14 newtons/cm²

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

Work out the volume of the cube.

$$p = 0.14 \text{ N/cm}^2$$

$$F = 56 \text{ N}$$

$$A = w \times w = w^2$$

$$0.14 = \frac{56}{w^2}$$

$$w^2 = \frac{56}{0.14}$$

$$w = \sqrt{400} \\ = 20$$

$$\text{Volume} = 20^3 = 8000 \text{ cm}^3$$

..... 8000 cm³

(Total for Question 13 is 4 marks)



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14 The diagram shows parallelogram $EFGH$.

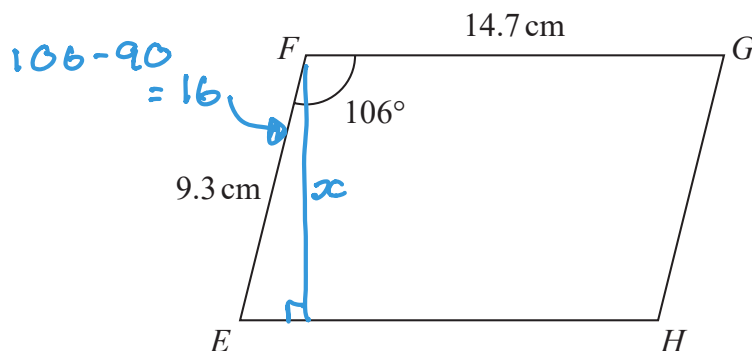


Diagram NOT accurately drawn

$EF = 9.3$ cm
 $FG = 14.7$ cm
 Angle $EFG = 106^\circ$

- (a) Work out the area of the parallelogram.
 Give your answer correct to 3 significant figures.

$$\cos 16 = \frac{x}{9.3} \quad x = 9.3 \times \cos 16$$

$$= 8.9397\dots$$

$$\text{area} = 8.9397\dots \times 14.7$$

$$= 131.414\dots$$

..... 131 cm²
 (2)

- (b) Work out the length of the diagonal EG of the parallelogram.
 Give your answer correct to 3 significant figures.

$$EG^2 = 9.3^2 + 14.7^2 - 2 \times 9.3 \times 14.7 \times \cos 106$$

$$= 377.944\dots$$

$$EG = \sqrt{377.944\dots}$$

$$= 19.4408\dots$$

↑
(3sf)

..... 19.4 cm
 (3)

(Total for Question 14 is 5 marks)



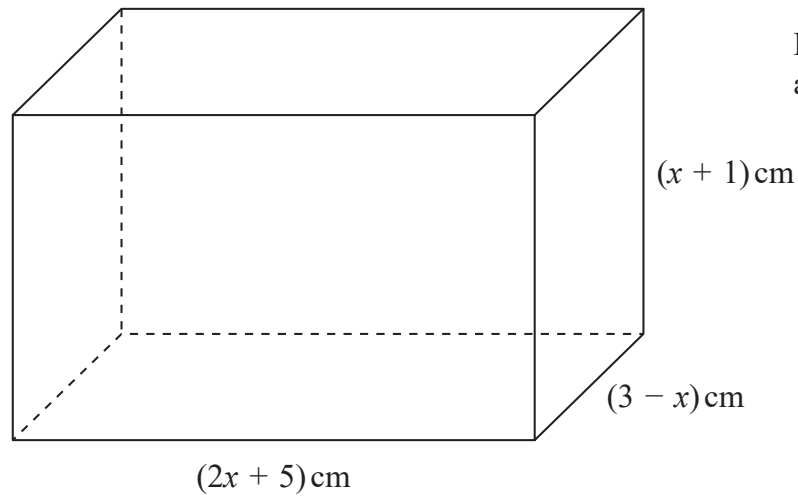


Diagram **NOT**
accurately drawn

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The diagram shows a cuboid of volume $V \text{ cm}^3$

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

$$\begin{aligned}
 V &= (2x + 5)(3 - x)(x + 1) \\
 &= (2x + 5)(6x + 15 - 2x^2 - 5x) \\
 &= (2x + 5)(15 - 2x^2 + x) \\
 &= 15x - 2x^3 + x^2 + 15 - 2x^2 + x \\
 &= 15 + 16x - x^2 - 2x^3 \\
 &\quad \text{as required.}
 \end{aligned}$$

(3)



There is a value of x for which the volume of the cuboid is a maximum.

(b) Find this value of x .

Show your working clearly.

Give your answer correct to 3 significant figures.

$$\text{Using } V = 15 + 16x - x^2 - 2x^3$$

$$\frac{dV}{dx} = 16 - 2x - 6x^2$$

$$16 - 2x - 6x^2 = 0$$

$$6x^2 + 2x - 16 = 0 \quad a = 6 \quad b = 2 \quad c = -16$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 6 \times (-16)}}{2 \times 6}$$

$$= \frac{-2 \pm \sqrt{4 + 384}}{12}$$

$$\frac{-2 + \sqrt{388}}{12}$$

$$\frac{-2 - \sqrt{388}}{12}$$

$$x = 1.47$$

(5)

$$= 1.4748\dots$$

not valid

(Total for Question 15 is 8 marks)

↑
(3sf)



$$16 \quad P = \frac{2a - c}{d}$$

$a = 58.4$ correct to 3 significant figures.

$c = 20$ correct to 2 significant figures.

$d = 3.6$ correct to 2 significant figures.

Work out the upper bound for the value of P .

Show your working clearly.

Give your answer correct to 2 decimal places.

$$a = 58.4 \begin{array}{l} \nearrow 58.45 \\ \searrow 58.35 \end{array}$$

$$c = 20 \begin{array}{l} \nearrow 20.5 \\ \searrow 19.5 \end{array}$$

$$d = 3.6 \begin{array}{l} \nearrow 3.65 \\ \searrow 3.55 \end{array}$$

$$P_{UB} = \frac{2 \times 58.45 - 19.5}{3.55}$$

$$= 27.4366\dots$$

↑
(2dp)

27.44

(Total for Question 16 is 3 marks)



17 (a) Show that $(6 + 2\sqrt{12})^2 = 12(7 + 4\sqrt{3})$

Show each stage of your working.

$$\begin{aligned}
 & (6 + 2\sqrt{12})(6 + 2\sqrt{12}) \\
 &= 36 + 12\sqrt{12} + 12\sqrt{12} + 4\sqrt{12}\sqrt{12} \\
 &= 36 + 24\sqrt{12} + 4 \times 12 \\
 &= 36 + 48 + 24\sqrt{12} \\
 &= 84 + 24\sqrt{12} \qquad \sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \\
 &= 84 + 24 \times 2\sqrt{3} \\
 &= 84 + 48\sqrt{3} = 12(7 + 4\sqrt{3}) \text{ as required.}
 \end{aligned}$$

(3)

(b) Simplify fully $\left(\frac{27a^{12}}{t^{15}}\right)^{\frac{2}{3}}$

$$\begin{aligned}
 & \left(\frac{t^{15}}{27a^{12}}\right)^{\frac{2}{3}} \\
 & \frac{t^{15 \times \frac{2}{3}}}{(\sqrt[3]{27})^2 a^{12 \times \frac{2}{3}}} = \frac{t^{10}}{9a^8}
 \end{aligned}$$

(3)

(Total for Question 17 is 6 marks)



18 There are 16 sweets in a bowl.

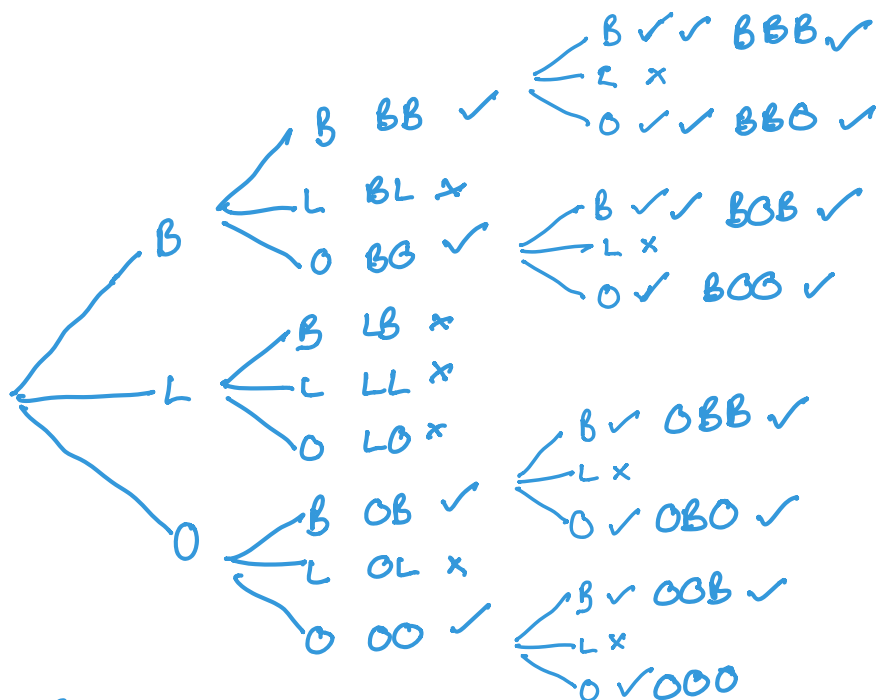
4 of the sweets are blackcurrant.

5 of the sweets are lemon.

7 of the sweets are orange.

Anna, Ravi and Sam each take at random one sweet from the bowl.

Work out the probability that the 5 lemon sweets are still in the bowl.



B L O
4 5 7

$$\frac{4}{16} \times \frac{3}{15} \times \frac{2}{14} + \frac{4}{16} \times \frac{3}{15} \times \frac{7}{14} + \frac{4}{16} \times \frac{7}{15} \times \frac{3}{14} + \frac{4}{16} \times \frac{7}{15} \times \frac{6}{14}$$

$$+ \frac{7}{16} \times \frac{4}{15} \times \frac{3}{14} + \frac{7}{16} \times \frac{4}{15} \times \frac{6}{14} + \frac{7}{16} \times \frac{6}{15} \times \frac{4}{14} + \frac{7}{16} \times \frac{6}{15} \times \frac{5}{14}$$

$$\frac{24 + 84 + 84 + 168 + 84 + 168 + 168 + 210}{3360}$$

$$\frac{990}{3360}$$

(Total for Question 18 is 4 marks)



19 The diagram shows a cuboid $ABCDEFGH$.

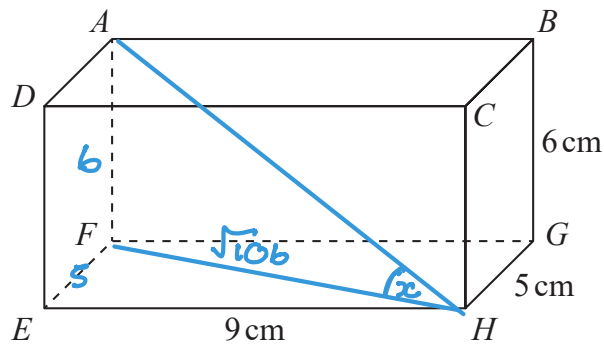


Diagram **NOT** accurately drawn

$EH = 9$ cm, $HG = 5$ cm and $GB = 6$ cm.

Work out the size of the angle between AH and the plane $EFGH$.
Give your answer correct to 3 significant figures.

$$HF = \sqrt{9^2 + 5^2}$$

$$= \sqrt{106}$$

$$\tan x = \frac{6}{\sqrt{106}}$$

$$x = \tan^{-1} \frac{6}{\sqrt{106}}$$

$$= 30.232 \dots$$

↑
3.s.f

30.2

(Total for Question 19 is 4 marks)



20 The curve C has equation $y = 4(x - 1)^2 - a$ where $a > 4$

Using the axes below, sketch the curve C.
On your sketch show clearly, in terms of a ,

- the coordinates of any points of intersection of C with the coordinate axes,
- the coordinates of the turning point.

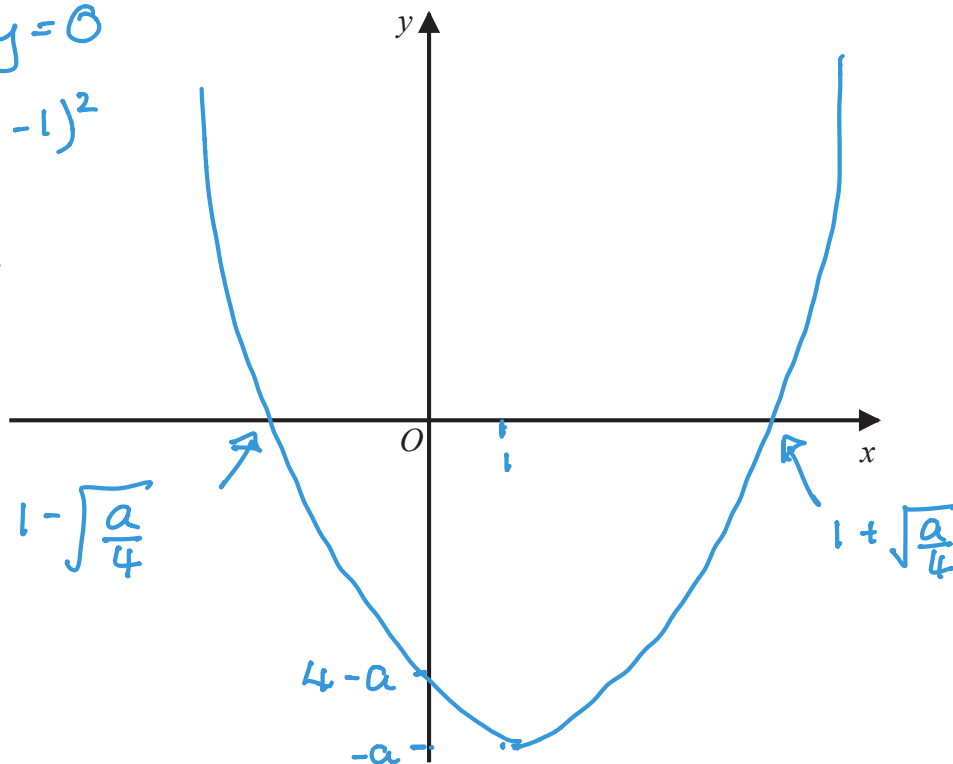
$$y = 4(x-1)^2 - a$$

turning point $\Rightarrow x = 1$
 $y = 4(1-1)^2 - a$
 $= -a$

y-axis $x = 0$
 $y = 4 - a$

x-axis $y = 0$
 $\frac{a}{4} = (x-1)^2$

$$1 \pm \sqrt{\frac{a}{4}} = x$$



(Total for Question 20 is 4 marks)



21 The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geq -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \dots$

$$\begin{aligned} fg(x) &= (x+3)^2 - 2(x+3) \\ &= x^2 + 6x + 9 - 2x - 6 \\ \therefore h(x) &= x^2 + 4x + 3 \end{aligned}$$

$$\begin{aligned} \underline{h^{-1}(x)} \quad y &= x^2 + 4x + 3 \\ &= (x+2)^2 - 4 + 3 \\ &= (x+2)^2 - 1 \end{aligned}$$

$$\begin{aligned} y+1 &= (x+2)^2 \\ x+2 &= \sqrt{y+1} \\ x &= -2 \pm \sqrt{y+1} \end{aligned}$$

$$\therefore h^{-1}(x) = -2 \pm \sqrt{x+1}$$

but $x \geq -2$

so $h^{-1}(x) = -2 + \sqrt{x+1}$

$$h^{-1}(x) = \dots -2 + \sqrt{x+1} \dots$$

(Total for Question 21 is 5 marks)



22 Triangle HJK is isosceles with $HJ = HK$ and $JK = \sqrt{80}$

H is the point with coordinates $(-4, 1)$

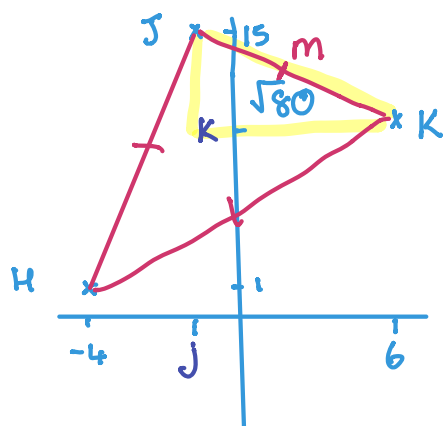
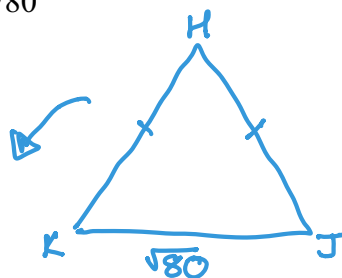
J is the point with coordinates $(j, 15)$ where $j < 0$

K is the point with coordinates $(6, k)$

M is the midpoint of JK .

The gradient of HM is 2

Find the value of j and the value of k .



gradient of $HM = 2$
so gradient $JK = -\frac{1}{2}$

$$\frac{JK}{JK} \quad \frac{k-15}{6-j} = -\frac{1}{2}$$

$$2(k-15) = -(6-j)$$

$$2k - 30 = -6 + j$$

$$2k - j = 24$$

$$2k - 24 = j \quad (1)$$

$$(\sqrt{80})^2 = (6-j)^2 + (15-k)^2$$

$$80 = 36 - 12j + j^2 + 225 - 30k + k^2$$

$$= j^2 - 12j + k^2 - 30k + 181$$

$$0 = (2k - 24)^2 - 12(2k - 24) + k^2 - 30k + 181$$

$$= 4k^2 - 96k + 576 - 24k + 288 + k^2 - 30k + 181$$

$$0 = 5k^2 - 150k + 1045$$



∴5

$$0 = k^2 - 30k + 209$$

11, 19

$$= (k - 11)(k - 19)$$

$$k = 11 \quad k = 19.$$

$$j = 2 \times 11 - 24 \\ = -2$$

$$j = 2 \times 19 - 24 \\ = 14$$

not valid as $j < 0$

$$j = \dots -2 \dots$$

$$k = \dots 11 \dots$$

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

